OLM 8.4. Calculation of the optimal clutch size in the great tit population of Wytham Woods

In Ch8Tradeoff we presented the results of a long-term study of great tits (*Parus major*) in Wytham Woods, Oxford (TBE Figure 8.19, p.157). We concluded that the close correspondence of the optimal clutch size calculated from the number (n) and the survival rate (s) of nestlings and the mean clutch size in the population observed for 19-years is due to the n–s trade-off, that is, to the nestling provisioning capacity of the feeding parental birds. We calculate the optimal clutch size in this OLM.

Data from regular nest box studies show (Figure 8.19) that

$$s = 0{,}18 - 0{,}01n\,, \tag{8.4.1}$$

where n is the number of hatchlings in a clutch and s is the probability that a hatchling is recaptured 3 months later. The points on the graphs of Figure 8.19 represent aggregated data for 19 years. As hatchling success is close to 1, n is considered to be the clutch size (number of eggs in a clutch) in the optimization model.

Since aging can be ignored in case of great tits (and many other bird species) and the only trade-off considered is the one represented by Eq. (8.4.1) the fitness function is greatly simplified in the optimisation model. The *pgr* of a non-aging, synchronously reproducing population is

$$\lambda = m + p = ns + p, \tag{8.4.2}$$

where m is the effective fecundity and p is the adult survival rate from year to year. This measure is explained in TBox 3.1. (p.37-38). However, assuming that the adult survival is independent of effective fecundity, the fitness function to be maximised can be simplified further, to yield

$$\lambda' = ns\,. \tag{8.4.3}$$

Substitution of Eq. (8.4.1) into Eq. (8.4.3) gives

$$\lambda' = \widetilde{\lambda}' = n(0{,}18 - 0{,}01n) = 0{,}18n - 0{,}01n^2\,. \tag{8.4.4}$$

This function has a single independent variable due to the trade-off (i.e. constraint; compare this equation to Eq. (8.7); p.152). The optimal clutch size n_{opt} at which fitness is maximal can be found by differentiating $\widetilde{\lambda}'$ with respect to n. The slope of the tangent (the first derivative) of a continuous function is zero at its maximal value (see also Figure 8.19.b), thus

$$\frac{\mathrm{d}\tilde{\lambda}'}{\mathrm{d}n} = 0{,}18 - 0{,}02n_{\mathrm{opt}} = 0. \tag{8.4.5}$$

It follows, that

$$n_{\mathrm{opt}} = \frac{0{,}18}{0{,}02} = 9. \tag{8.4.6}$$

References

Bell, G. (1997). Selection: the mechanism of evolution. New York, Springer.
Savill, P., Perrins, C., Fisher, N. and Kirby, K. (2011). Wytham Woods: Oxford's Ecological Laboratory. Oxford University Press.