OLM 2.2. The stochastic process of dying

In TBox 2.1 we presented the discrete and the continuous description of the stochastic process of survival and dying. Because of the centrality of the issue, here we elaborate more on the consistencies in Table 2.1 and their relation to the summation of a geometric series.

As presented in Table 2.1, probability of survival until time t (i.e. until the end of the t^{th} day) is p^t. The probability of dying *exactly* on day t can be calculated in two ways. It is either the difference of probability of being alive at time t-1 and at time t, or the product of t-1 consecutive survivals and the probability of dying at day t. The results are identical:

$$p^{t-1} - p^t = p^{t-1}(1-p). \tag{2.2.1}$$

The probability of the event of *not* surviving to time t is 1-p^t. This means that the individual had died either on day t, or some day earlier. Since the event of dying on a certain day precludes dying on another, the probability of not reaching day t alive is the sum of the probabilities of death on each day up to t:

$$1 - p^t = (1-p) + p(1-p) + p^2(1-p) + \ldots + p^{t-1}(1-p), \tag{2.2.2}$$

or, by dividing with 1-p,

$$1 + p + p^2 + \ldots + p^{t-1} = \frac{1-p^t}{1-p}, \tag{2.2.3}$$

which is the well-known summation formula for geometric series (http://mathworld.wolfram.com/GeometricSeries.html).

In the limit $t \to \infty$ the probability of survival goes to 0 (i.e., $p^t \to 0$ for $0 \le p < 1$) and the probability of ultimate death goes to 1. Eventually, our individual will die for sure. (Probabilists would say 'almost sure', because they are precise enough to distinguish between a probability 0 event and the impossible (TBox 11.1); however it does not really matter for our individual in question.) In this limit Eqs. (2.2.2) and (2.2.3) become the infinite sums

$$1 = (1-p) + p(1-p) + p^2(1-p) + \ldots \tag{2.2.4}$$

and

$$1 + p + p^2 + \ldots = \frac{1}{1-p}. \tag{2.2.5}$$

Again, Eq. (2.2.5) is the well-known summation formula for infinite geometric series.