

OLM10.6. An example for the calculation of niche overlap

We have shown in TBox 10.2 (p. 211-212) how niche overlap can be calculated from the impact and sensitivity functions. To this end, it is indispensable to specify the niche space, which is not an easy task usually. Fortunately, niche overlap can be estimated from the parameters of the implicit models of the dynamics as well. The niche overlap thus estimated and the correlations of the differences between the species may be of help in exploring the niche space, too. For example, Godoy and Levine (2014) have deployed this method in proving the importance of temporal segregation of flowering times in annual plant communities.

The four-step scheme below makes the calculation of Pianka's niche overlap index (TBox 9.1, p. 172) possible from the parameters of an arbitrarily complicated implicit dynamics (e.g. TBox 10.2, p. 211):

Step 1: Write down the equations expressing the density dependence of the growth rates (r or λ). The number of equations in the system is equal to the number of species (alleles, clones); in order to keep the treatment simple, we show the two-species case which can be generalized.

Step 2: Calculate the elements of the community matrix (Eq. 9.22): these are the partial derivatives of the growth rates with respect to population densities, evaluated at the fixed point (linearization, TBox 1.1, p. 9):

$$a_{ij} = -\frac{\partial r_i}{\partial N_j}(\hat{N}_i, \hat{N}_j) = -\frac{\partial \ln \lambda_i}{\partial N_j}(\hat{N}_i, \hat{N}_j) \quad (10.6.1)$$

Step 3: Make use of the fact that $r_i=0$, or $\lambda_i=1$ at the fixed point.

Step 4: Calculate Pianka's niche overlap from the community matrix entries (Eq.9.8, Eq.10.10; (Pianka 1973, see also Chesson 2013).

Let us apply these four steps in a particular example.

Step 1: The following model is often used to describe competition between annual plant species (e.g. Godoy et al. 2014, Godoy and Levine 2014, Kraft et al. 2015):

$$\lambda_i = \frac{N_{i,t+1}}{N_{i,t}} = (1 - g_i)s_i + g_i F_i = (1 - g_i)s_i + g_i \frac{F_{0i}}{1 + \alpha_{ii}g_i N_{i,t} + \alpha_{ij}g_j N_{j,t}}, \quad (10.6.2)$$

where $N_{i,t}$ is the size of the population of species i after seed dispersal (the number of seeds in the soil) at the end of the t -th vegetation period; g_i is the proportion of seeds germinating in species i ; s_i is the survival rate of non-germinating seeds; $F_{i,t}$ is the average seed yield per germinated seed of species i at year t ; F_{0i} is the same yield without competition; λ_{ij} is the parameter specifying the strength of the effect of the density (= the number of germinated seeds) of species j on the seed yield of species i .

The model is built on the assumption that germinated seeds compete for resources, and competition reduces the average number of seeds produced by a germinated seed (also considering the zero contribution of those which germinated but died before seed set). Since competition occurs only among germinated seeds, we conjecture that niche overlap depends only on the per capita yield reduction due to the density of germinated seeds. Now let us see if this conjecture is right.

Step 2: We determine the entries of the community matrix as the partial derivatives of the $\ln \lambda$ functions at the fixed point:

$$a_{ij} = - \left. \frac{\partial \ln \lambda_i}{\partial N_j} \right|_{\hat{N}_i, \hat{N}_j} = - \frac{1}{\lambda_i(\hat{N}_i, \hat{N}_j)} \left. \frac{\partial \lambda_i}{\partial N_j} \right|_{\hat{N}_i, \hat{N}_j} = - \left. \frac{\partial \lambda_i}{\partial N_j} \right|_{\hat{N}_i, \hat{N}_j} \quad (10.6.3)$$

Rearrangement of the formulas relies on that $\lambda=1$ at equilibrium. Now we apply this general relation to the particular case we are studying:

$$\left. \frac{\partial \lambda_i}{\partial N_j} \right|_{\hat{N}_i, \hat{N}_j} = -g_i \frac{F_{0i}}{(1 + \alpha_{ii}g_i \hat{N}_{i,t} + \alpha_{ij}g_j \hat{N}_{j,t})^2} \alpha_{ij} g_j \quad (10.6.4)$$

Step 3: With $\lambda_i=1$:

$$(1 - g_i)s_i + g_i \frac{F_{0i}}{1 + \alpha_{ii}g_i \hat{N}_{i,t} + \alpha_{ij}g_j \hat{N}_{j,t}} = 1 \quad (10.6.5)$$

$$g_i \frac{F_{0i}}{1 + \alpha_{ii}g_i \hat{N}_{i,t} + \alpha_{ij}g_j \hat{N}_{j,t}} = 1 - (1 - g_i)s_i = \beta_i \quad (10.6.6)$$

Substituting this into Eq. (10.6.3):

$$\left. \frac{\partial \lambda_i}{\partial N_j} \right|_{\hat{N}_i, \hat{N}_j} = - \frac{\beta_i^2 g_j}{g_i F_i} \alpha_{ij} \quad (10.6.7)$$

Step 4: The Pianka's niche overlap index calculated from the entries of the community matrix is

$$\rho = \sqrt{\frac{a_{ij}a_{ji}}{a_{ii}a_{jj}}} = \sqrt{\frac{\frac{\beta_i^2 g_j}{g_i F_{0i}} \alpha_{ij} \frac{\beta_j^2 g_i}{g_j F_{0j}} \alpha_{ji}}{\frac{\beta_i^2 g_i}{g_i F_{0i}} \alpha_{ii} \frac{\beta_j^2 g_j}{g_j F_{0j}} \alpha_{jj}}} = \sqrt{\frac{\alpha_{ij} \alpha_{ji}}{\alpha_{ii} \alpha_{jj}}} \quad (10.6.8)$$

That is, the niche overlap does not depend on the other parameters of the dynamics in this model. Our conjecture has proven correct: niche overlap is higher if the yield of one species depends more on that of others compared to its own yield. According to the findings of Godoy and Levine (2014) phenological differences decreased the niche overlap estimated with the model above, providing evidence for the importance of phenology in the coexistence of the annual species they studied. Thus, the niche segregation of these species is temporal.

References

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