# OLM 10.1. Discrete and continuous niche: the formal problem

Resource types can be discrete categories (like "phosphate" and "silicate", or seed categories of three different hardnesses) or they may constitute a continuum of qualities (like a continuum of seed hardness). In TBox 10.1 we explained that the two cases are not fundamentally different from biological point of view. Still, the mathematical descriptions of the two cases appear to be very different. The concentrations of the finitely many resource types play the role of regulating variables in the discrete case. However, the continuous case corresponds to infinitely many regulating variables, which sounds quite mystical. What does it mean?

Let us start with the differing formalism. The finitely many concentrations of the discrete case are usually collected into a vector: $\boldsymbol{R} = (R_1, R_2, \dots, R_D)$, where $D$ is the number of resource types – and the dimension of the vector. In contrast, we specify the concentrations as a function of a continuous $q$ (quality) variable in the continuous case: $R(q)$ is the concentration of seeds of hardness $q$. More precisely: $R(q)\Delta q$ is the concentration of seeds within the $(q, q + \Delta q)$ quality interval, if $\Delta q$ is sufficiently small (cf. differentiation, TB 1.1).

We used to consider vectors and functions, as quite different mathematical constructs. However, upon inspecting these two cases more thoroughly we discover that they are the same in that every resource quality has a concentration assigned to it. We may consider the $\boldsymbol{R}$ vector of the discrete case as a function on the "index" set $\{1,2, \dots, D\}$ which assigns the $R_i$ value to the element *i* of the index set – and it could be labeled by $R(i)$ with equal legitimacy. Conversely, the function $R(q)$ could be transcribed into the vector notation $R_q$ if the continuous $q$ variable was regarded as the index of the vector; the feasible values of $q$ constitute the index set then. Of course the vector we deal with has infinitely many components in this case.

Two- or three-component vectors are usually visualized as arrows in the geometric plane or space. We are also comfortable with the notions of 4 or *D*-component vectors as arrows in 4 or *D*-dimensional 'abstract' spaces. In the same vein we may talk about the $R(q)$ function as a vector of an infinite dimensional (*D*=∞) vector/function space. We need not bother too much about imagining these abstract spaces. They are just convenient expressions for the fact that we deal with four, or infinitely many, variables. Still, our intuition is not completely hapless with them, either. It is reassuring to know that any 1, 2 or 3 dimensional subspace of a higher (even infinite) dimensional space is just like our ordinary 1, 2, or 3 dimensional geometric space. If we have four variables, we can plot any two of them on a sheet of paper. We do cheat the Reader in TBox 10.2, when we discuss the area of a parallelogram spanned by two vectors, dimensionality of which could be anything, including infinite. We just mean it within the two dimensional plane defined by the vectors, and apply elementary plane geometry.

Well, infinite dimension is not *that* simple. It raises many conceptual problems which are out of our scope here and which render "functional analysis" a difficult discipline (Kreyszig 2007). How should we define the length of an infinite dimensional vector, for example? Fortunately, the fundamental issues have been taken care of by mathematicians. For ecology we need to know about infinite dimensional spaces only in order to avoid the confusion of our ecological intuition by the purely formal problem of a resource continuum being an infinite number of different resources. Keep in mind, that you discretize the seed hardness continuum in your empirical practice, anyhow. Feel free to change sums to integrals, or back, when appropriate (e.g. Eqs. (10.1), (10.2)). Ask somebody, when unsure.

Needless to say, that the very same problem of discrete/continuous description may arise considering any regulating factor, not just resources, as it may arise also in spatial and temporal niche segregation context (Figure 10.1). We identified the niche space with the index set of the regulating variables. In discrete case the finite $D$ number of regulating variables are collected into a D-dimensional vector; the $D$ elements of the index set corresponds to $D$ "point" of the niche space. (Why do we refer to a finite set, as a 'space'? Just because we want to save the biological intuition attached to the term 'niche space' by Hutchinson.) In the continuous case $D=\infty$ the infinitely many regulating variables forms an infinite dimensional vector. The niche space, or the index set, is a 'true', continuous (Euclidean) space – probably a finite dimensional one – spanned by the continuous index variables, as seed hardness.

## References

Kreyszig, E. (2007). *Introductory Functional Analysis with Applications*, Wiley India Pvt. Limited.